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# Lifetime Ratios of Beauty Hadrons at the Next-to-Leading Order in QCD

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## Abstract

We compute the next-to-leading order QCD corrections to spectator effects in the lifetime ratios of beauty hadrons. With respect to previous calculations, we take into account the non vanishing value of the charm quark mass. We obtain the predictions  $\tau(B^+)/\tau(B_d) = 1.06 \pm 0.02$ ,  $\tau(B_s)/\tau(B_d) = 1.00 \pm 0.01$  and  $\tau(\Lambda_b)/\tau(B_d) = 0.90 \pm 0.05$ , in good agreement with the experimental results. In the case of  $\tau(B_s)/\tau(B_d)$  and  $\tau(\Lambda_b)/\tau(B_d)$ , however, some contributions, which either vanish in the vacuum insertion approximation or represent a pure NLO corrections, have not been determined yet.

# 1 Introduction

The inclusive decay rates of beauty hadrons can be computed by expanding the amplitudes in increasing powers of  $\Lambda_{QCD}/m_b$  [1, 2] and using the assumption of quark-hadron duality. The leading term in this expansion reproduces the predictions of the naïve quark spectator model, and the first correction of  $\mathcal{O}(1/m_b)$  is absent in this case.

Within this theoretical framework, up to terms of  $\mathcal{O}(1/m_b^2)$  only the  $b$  quark enters the short-distance weak decay, while the light quarks in the hadron interact through soft gluons only. For this reason, the lifetime ratios of beauty hadrons are predicted to be unity at this order, with corrections which are at most of few percent.

Spectator contributions, which are expected to be mainly responsible for the lifetime differences of beauty hadrons, only appear at  $\mathcal{O}(1/m_b^3)$  in the Heavy Quark Expansion (HQE). These effects, although suppressed by powers of  $1/m_b$ , are enhanced, with respect to leading contributions, by a phase-space factor of  $16\pi^2$ , being  $2 \rightarrow 2$  processes instead of  $1 \rightarrow 3$  decays. Indeed, the inclusion of these corrections at the leading order (LO) in QCD has allowed to reproduce the observed pattern of the lifetime ratios [3, 4].

In this paper we compute the next-to-leading order (NLO) QCD corrections to spectator effects in the lifetime ratios of beauty hadrons. In the limit of vanishing charm quark mass, these corrections have been already computed in ref. [5], and we refer to this paper for many details of the NLO calculation. We find that the inclusion of the NLO corrections improves the agreement with the experimental measurements. Moreover, it increases the accuracy of the theoretical predictions by reducing significantly the dependence on the operator renormalization scale.

After our calculation was completed, the NLO corrections to spectator effects in the ratio  $\tau(B^+)/\tau(B_d)$  have been also presented in ref. [6]. We agree with their results. With respect to ref. [6], we also perform in this paper the NLO calculation of the Wilson coefficients entering the HQE of the ratios  $\tau(B_s)/\tau(B_d)$  and  $\tau(\Lambda_b)/\tau(B_d)$ .

By using the lattice determinations of the relevant hadronic matrix elements [7]-[10], we have performed a theoretical estimate of the lifetime ratios and obtained the NLO predictions

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.06 \pm 0.02, \quad \frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.90 \pm 0.05. \quad (1)$$

These estimates must be compared with the experimental measurements [11]

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.074 \pm 0.014, \quad \frac{\tau(B_s)}{\tau(B_d)} = 0.948 \pm 0.038, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.796 \pm 0.052. \quad (2)$$

The theoretical predictions turn out to be in good agreement with the experimental data. As we will discuss below, however, in the case of  $\tau(B_s)/\tau(B_d)$  and  $\tau(\Lambda_b)/\tau(B_d)$  the theoretical predictions can be further improved, since some contributions, which either vanish in the vacuum insertion approximation (VIA) or represent a pure NLO correction, are still lacking. With respect to the results of ref. [5], we find that the NLO charm quark mass corrections are rather large for some of the Wilson coefficients. Nevertheless, the total effect of these contributions on the lifetime ratios is numerically small.

An important check of the perturbative calculation performed in this paper is provided by the cancellation of the infrared (IR) divergences in the expressions of the Wilson coefficients, in spite these divergences appear in the individual amplitudes. The presence of these divergences explicitly shows that the Bloch-Nordsieck theorem does not apply in non-abelian gauge theories [12]-[14]. We have also checked that our results are explicitly gauge invariant and have the correct ultraviolet (UV) renormalization-scale dependence as predicted by the known LO anomalous dimensions of the relevant operators.

The HQE of the lifetime ratios results in a series of local operators of increasing dimension defined in the Heavy Quark Effective Theory (HQET). Indeed, renormalized operators in QCD mix with operators of lower dimension, with coefficients proportional to powers of the  $b$ -quark mass. In this case, the dimensional ordering of the HQE would be lost. In order to implement the expansion, the matrix elements of the local operators should be cut-off at a scale smaller than the  $b$ -quark mass, which is naturally realized in the HQET.<sup>1</sup> In the case of  $\tau(B^+)/\tau(B_d)$ , only flavour non-singlet operators enter the expansion. For these operators, the mixing with lower dimensional operators is absent, and the HQE can be expressed in terms of operators defined in QCD. For these operators, the matching between QCD and HQET has been computed, at the NLO, in ref. [5].

As mentioned before, the NLO predictions for the lifetime ratios  $\tau(B_s)/\tau(B_d)$  and  $\tau(\Lambda_b)/\tau(B_d)$  may still be improved in some ways. In particular:

- the contributions of the four-fermion operators containing the charm quark field and of the penguin operator (see eqs. (10) and (11)) have not been determined yet. These contributions, which either vanish in the VIA (charm operators) or represent a pure NLO correction (penguin operator), do not affect the theoretical determination of  $\tau(B^+)/\tau(B_d)$  and represent only an  $SU(3)$ -breaking effect for  $\tau(B_s)/\tau(B_d)$ ;
- the lattice determination of the hadronic matrix elements have been performed by neglecting penguin contractions (i.e. eye diagrams), which exist in the case of flavour singlet operators. Also in this case, the corresponding contributions cancel in the ratio  $\tau(B^+)/\tau(B_d)$  but affect  $\tau(\Lambda_b)/\tau(B_d)$  and  $\tau(B_s)/\tau(B_d)$ , the latter only through  $SU(3)$ -breaking effects;
- the NLO anomalous dimension of the four-fermion  $DB = 0$  operators in the HQET is still unknown. For this reason, the renormalization scale evolution of the  $\Lambda_b$  matrix elements, computed on the lattice in the HQET at a scale smaller than the  $b$ -quark mass [8, 9], has been only performed at the LO. For  $B$  mesons, a complete NLO evolution has been performed by using operators defined in QCD [10].

For these reasons, at present, the best theoretical accuracy is achieved in the determination of the lifetime ratio  $\tau(B^+)/\tau(B_d)$ .

We conclude this section by presenting the plan of this paper. In sect. 2 we review the basic formalism of the HQE applied to the lifetime ratios of beauty hadrons. Details of the NLO calculation and the numerical results obtained for the Wilson coefficients are given in sect. 3. In sect. 4 we present the predictions for the lifetime ratios of beauty hadrons.

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<sup>1</sup>On this point, we disagree with the general statement of ref. [6] according to which the HQE can be performed in QCD also in the presence of mixing with lower dimensional operators.

Finally, we collect in the appendix the analytical expressions for the Wilson coefficient functions.

## 2 HQE for the lifetime ratios of beauty hadrons

Using the optical theorem, the inclusive decay width  $\Gamma(H_b)$  of a hadron containing a  $b$  quark can be written as

$$\Gamma(H_b) = \frac{1}{M_{H_b}} \text{Im} \langle H_b | \mathcal{T} | H_b \rangle, \quad (3)$$

where the transition operator  $\mathcal{T}$  is given by

$$\mathcal{T} = i \int d^4x T \left( \mathcal{H}_{eff}^{\Delta B=1}(x) \mathcal{H}_{eff}^{\Delta B=1}(0) \right) \quad (4)$$

and  $\mathcal{H}_{eff}^{\Delta B=1}$  is the effective weak hamiltonian which describes  $\Delta B = 1$  transitions.

By neglecting the Cabibbo suppressed contribution of  $b \rightarrow u$  transitions ( $|V_{ub}|^2/|V_{cb}|^2 \sim 0.16\lambda^2$ ) and terms proportional to  $|V_{td}|/|V_{ts}|$  in the penguin sector, the  $\Delta B = 1$  effective hamiltonian can be written in the form

$$\mathcal{H}_{eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} V_{cb}^* \left[ C_1 (Q_1 + Q_1^c) + C_2 (Q_2 + Q_2^c) + \sum_{i=3}^6 C_i Q_i + C_{8G} Q_{8G} + \sum_{l=e,\mu,\tau} Q_l \right] + h.c.. \quad (5)$$

The  $C_i$  are the Wilson coefficients, known at the NLO in perturbation theory [15]-[17], and the operators  $Q_i$  are defined as

$$\begin{aligned} Q_1 &= (\bar{b}_i c_j)_{V-A} (\bar{u}_j d'_i)_{V-A}, & Q_2 &= (\bar{b}_i c_i)_{V-A} (\bar{u}_j d'_j)_{V-A}, \\ Q_1^c &= (\bar{b}_i c_j)_{V-A} (\bar{c}_j s'_i)_{V-A}, & Q_2^c &= (\bar{b}_i c_i)_{V-A} (\bar{c}_j s'_j)_{V-A}, \\ Q_3 &= (\bar{b}_i s_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, & Q_4 &= (\bar{b}_i s_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \\ Q_5 &= (\bar{b}_i s_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, & Q_6 &= (\bar{b}_i s_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \\ Q_{8G} &= \frac{g_s}{8\pi^2} m_b \bar{b}_i \sigma^{\mu\nu} (1 - \gamma^5) t_{ij}^a s_j G_{\mu\nu}^a, & Q_l &= (\bar{b}_i c_i)_{V-A} (\bar{\nu}_l l)_{V-A}, \end{aligned} \quad (6)$$

where  $d' = \cos \theta_c d + \sin \theta_c s$  and  $s' = -\sin \theta_c d + \cos \theta_c s$  ( $\theta_c$  is the Cabibbo angle). Here and in the following we use the notation  $(\bar{q}q)_{V\pm A} = \bar{q} \gamma_\mu (1 \pm \gamma_5) q$  and  $(\bar{q}q)_{S\pm P} = \bar{q} (1 \pm \gamma_5) q$ . A sum over repeated colour indices is always understood.

Because of the large mass of the  $b$  quark, it is possible to construct an Operator Product Expansion (OPE) for the transition operator  $\mathcal{T}$  of eq. (4), which results in a sum of local operators of increasing dimension [1, 2]. We include in this expansion terms up to  $\mathcal{O}(1/m_b^2)$  plus those  $1/m_b^3$  corrections that come from spectator effects and are enhanced by the phase space. The resulting expression for the inclusive width of eq. (3) is given by

$$\Gamma(H_b) = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left[ c^{(3)} \frac{\langle \bar{b}b \rangle_{H_b}}{2M_{H_b}} + c^{(5)} \frac{g_s}{m_b^2} \frac{\langle \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b \rangle_{H_b}}{2M_{H_b}} + \frac{96\pi^2}{m_b^3} \sum_k c_k^{(6)} \frac{\langle O_k^{(6)} \rangle_{H_b}}{2M_{H_b}} \right], \quad (7)$$

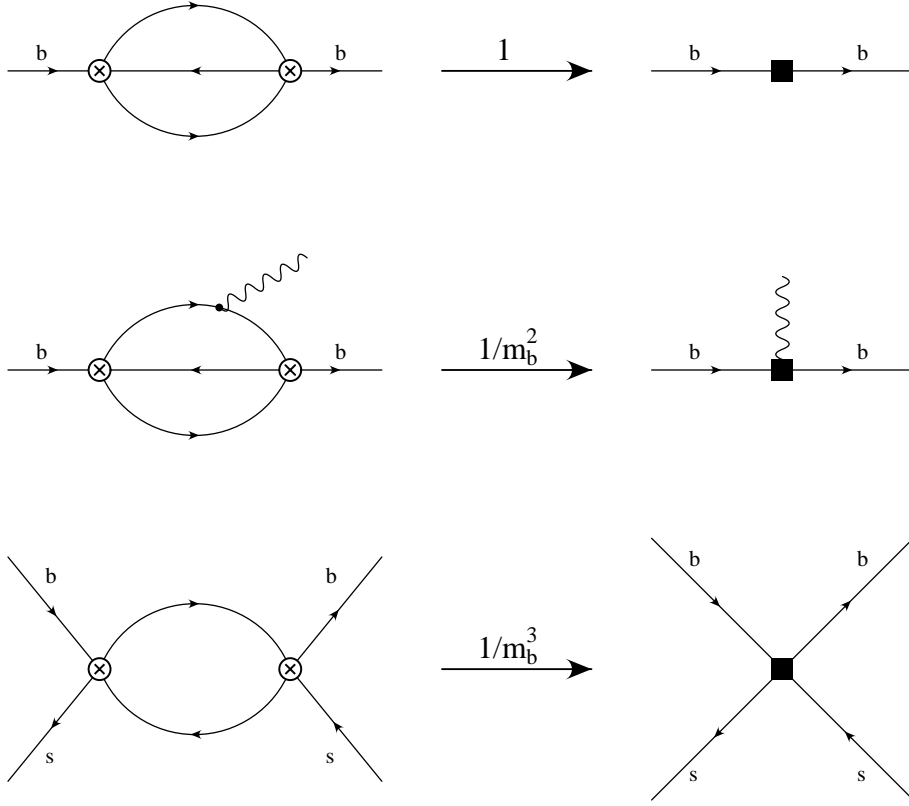


Figure 1: *Examples of LO contributions to the transition operator  $\mathcal{T}$  (left) and to the corresponding local operator (right). The crossed circles represent the insertions of the  $\Delta B = 1$  effective hamiltonian. The black squares represent the insertion of a  $\Delta B = 0$  operator.*

where  $\langle \cdots \rangle_{H_b}$  denotes the forward matrix element between two hadronic states  $H_b$ , defined with the covariant normalization. The operators  $O_k^{(6)}$  are a set of four-fermion, dimension-six operators to be specified below, which represent the contribution of hard spectator effects. At the lowest order in QCD, the diagrams entering the calculation of  $\Gamma(H_b)$  are shown in fig. 1.

The matrix elements of dimension-three and dimension-five operators, appearing in eq. (7), can be expanded by using the HQET

$$\begin{aligned} \langle \bar{b}b \rangle_{H_b} &= 2M_{H_b} \left( 1 - \frac{\mu_\pi^2(H_b) - \mu_G^2(H_b)}{2m_b^2} + \mathcal{O}(1/m_b^3) \right), \\ g_s \langle \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b \rangle_{H_b} &= 2M_{H_b} \left( 2\mu_G^2(H_b) + \mathcal{O}(1/m_b) \right). \end{aligned} \quad (8)$$

By using these expansions, we can finally compute, from eq. (7), the lifetime ratio of two beauty hadrons

$$\frac{\Gamma(H_b)}{\Gamma(H'_b)} = 1 - \frac{\mu_\pi^2(H_b) - \mu_\pi^2(H'_b)}{2m_b^2} + \left( \frac{1}{2} + \frac{2c^{(5)}}{c^{(3)}} \right) \frac{\mu_G^2(H_b) - \mu_G^2(H'_b)}{m_b^2} +$$

$$\frac{96\pi^2}{m_b^3 c^{(3)}} \sum_k c_k^{(6)} \left( \frac{\langle O_k^{(6)} \rangle_{H_b}}{2M_{H_b}} - \frac{\langle O_k^{(6)} \rangle_{H'_b}}{2M_{H'_b}} \right). \quad (9)$$

The dimension-six operators in eq. (9), which express the hard spectator contributions, are the four current-current operators<sup>2</sup>

$$\begin{aligned} \mathcal{O}_1^q &= (\bar{b}q)_{V-A} (\bar{q}b)_{V-A}, & \mathcal{O}_2^q &= (\bar{b}q)_{S-P} (\bar{q}b)_{S+P}, \\ \mathcal{O}_3^q &= (\bar{b}t^aq)_{V-A} (\bar{q}t^ab)_{V-A}, & \mathcal{O}_4^q &= (\bar{b}t^aq)_{S-P} (\bar{q}t^ab)_{S+P}, \end{aligned} \quad (10)$$

with  $q = u, d, s, c$ , and the penguin operator

$$\mathcal{O}_P = (\bar{b}t^ab)_V \sum_{q=u,d,s,c} (\bar{q}t^aq)_V. \quad (11)$$

In these definitions, the symbols  $b$  and  $\bar{b}$  denote the heavy quark fields in the HQET.

The Wilson coefficients  $c^{(3)}$  and  $c^{(5)}$ , of the dimension-three and dimension-five operators in eq. (9), have been computed at the LO in ref. [18], while the NLO corrections to  $c^{(3)}$  have been evaluated in [19]-[24]. The NLO corrections to  $c^{(5)}$  are still missing. Their numerical contribution to the lifetime ratios, however, is expected to be negligible.

The coefficient functions of the dimension-six current-current operators  $\mathcal{O}_k^q$ , have been computed at the LO in ref. [4, 25] for  $q = u, d, s$ , and in ref. [26] for  $q = c$ . The coefficient function of the penguin operator  $\mathcal{O}_P$  vanishes at the LO.

In this paper we have computed the NLO QCD corrections to the coefficient functions of the operators  $\mathcal{O}_k^q$  with  $q = u, d, s$ . The operators containing the charm quark fields contribute, as valence operators, only to the inclusive decay rate of  $B_c$  mesons, and their contribution to non-charmed hadron decay rates is expected to be negligible. The calculation of the NLO corrections to these coefficient functions, as well as the NLO calculation of the coefficient function of the penguin operator, has not been performed yet. The non-perturbative determinations of the corresponding matrix elements are also lacking at present. However, as we will discuss in sect. 4, these contributions only enter the theoretical estimates of the ratios  $\tau(B_s)/\tau(B_d)$  and  $\tau(\Lambda_b)/\tau(B_d)$ , and vanish in the VIA.

### 3 NLO calculation of the Wilson coefficients

All the details of the matching procedure used to determine the Wilson coefficients at the NLO have been given in ref. [5], where the calculation has been performed in the limit of vanishing charm quark mass. A finite value of the charm quark mass does not introduce conceptual difficulties in the calculation, besides requiring the evaluation of one- and two-loop integrals with an additional mass scale. For this reason, we only remind in this section the general strategy of the perturbative calculation, and refer the interested reader to ref. [5] for further details. In this section, we will also present the numerical results for the Wilson coefficients, while the analytical expressions of these coefficients are collected in the appendix.

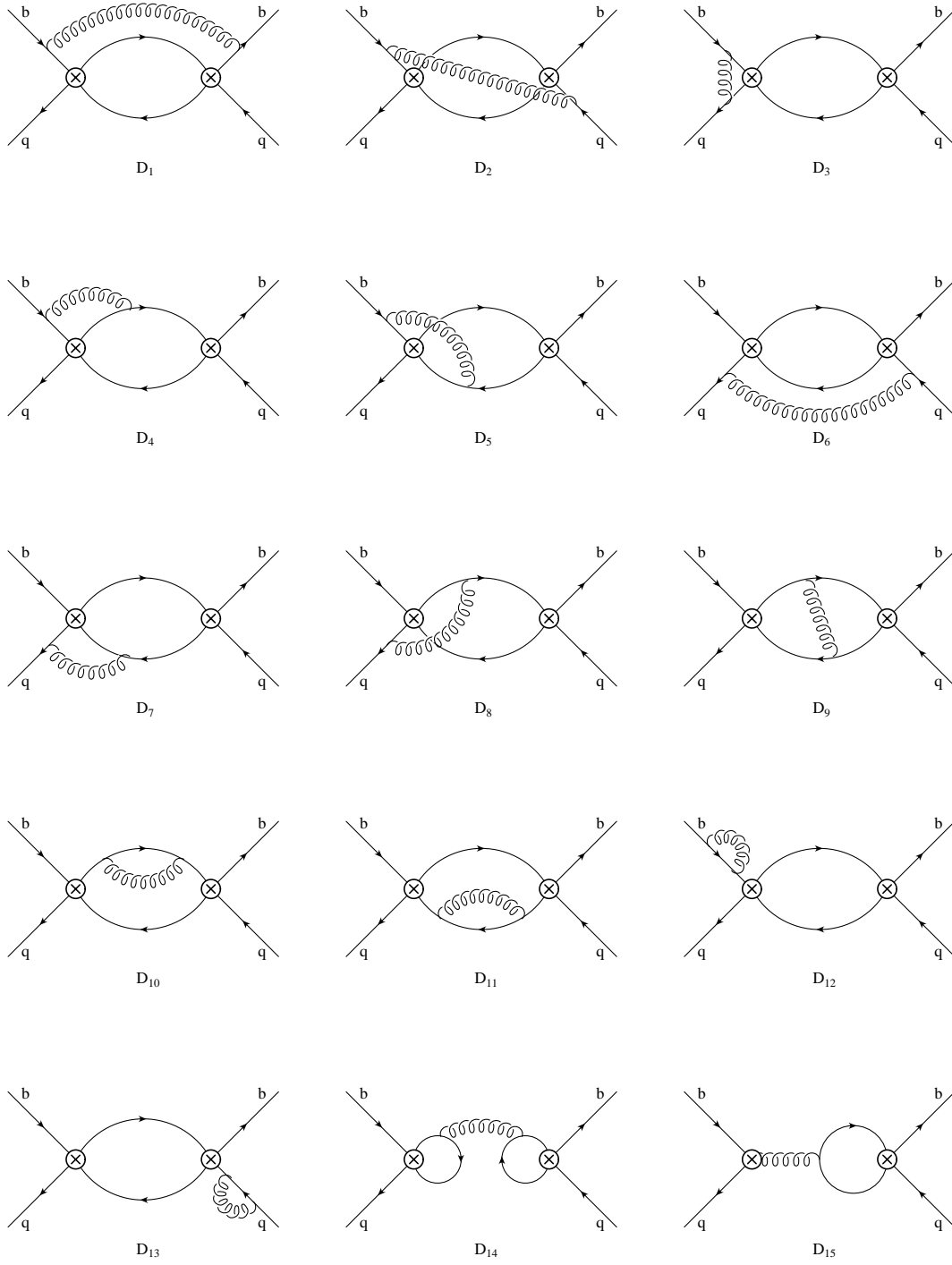


Figure 2: *Feynman diagrams which contribute at NLO to the matrix element of the transition operator  $\mathcal{T}$  in the case  $q = s$ . In the other cases,  $q = u, d$ , diagrams  $D_{14}$  and  $D_{15}$  are Cabibbo suppressed and have been neglected in the calculation.*

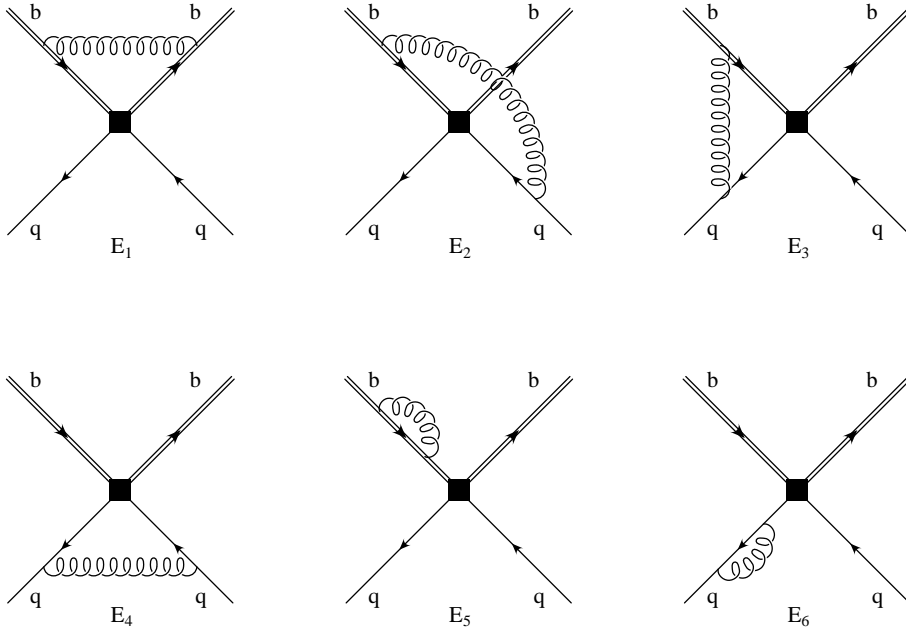


Figure 3: *Feynman diagrams which contribute, at NLO, to the matrix element of the  $\Delta B = 0$  operators entering the HQET.*

In order to compute the Wilson coefficients of the  $\Delta B = 0$  operators at the NLO, we have evaluated in QCD the imaginary part of the diagrams shown in fig. 2 (*full theory*) and in the HQET the diagrams shown in fig. 3 (*effective theory*). The external quark states have been taken on-shell and all quark masses, except  $m_b$  and  $m_c$ , have been neglected. More specifically, we have chosen the heavy quark momenta  $p_b^2 = m_b^2$  in QCD and  $k_b = 0$  in the HQET, and  $p_q = 0$  for the external light quarks. In this way, we automatically retain the leading term in the  $1/m_b$  expansion. We have performed the calculation in a generic covariant gauge, in order to check the gauge independence of the final results. Two-loop integrals have been reduced to a set of independent master integrals by using the recurrence relation technique [27]-[29] implemented in the TARCER package [30]. Equations of motion have been used to reduce the number of independent operators.

Some diagrams, both in the full and in the effective theory, are plagued by IR divergences. These divergences do not cancel in the final partonic amplitudes, and provide an example of violation of the Bloch-Nordsieck theorem in non-abelian gauge theories [12]-[14]. IR poles, however, are expected to cancel in the matching and we have explicitly checked that, in the computation of the coefficient functions, this cancellation takes place.

We use  $D$ -dimensional regularization with anticommuting  $\gamma_5$  (NDR) to regularize both UV and IR divergences. As discussed in details in ref. [5], the presence of dimensionally-regularized IR divergences introduces subtleties in the matching procedure. The matching must be consistently performed in  $D$  dimensions. This requires, in particular, enlarging

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<sup>2</sup>Note that, with respect to ref. [5], we use a different basis of operators.



the operator basis to include (renormalized) evanescent operators, which must be inserted in the one-loop diagrams of the effective theory. Because of the IR divergences, the matrix elements of the renormalized evanescent operators do not vanish in the  $D \rightarrow 4$  limit [31], and give a finite contribution in the matching procedure.

As a check of the perturbative calculation, we have verified that our results for the Wilson coefficients satisfy the following requirements:

- *gauge invariance*: the coefficient functions in the  $\overline{\text{MS}}$  scheme are explicitly gauge-invariant. The same is true for the full and the effective amplitudes separately;
- *renormalization-scale dependence*: the coefficient functions have the correct logarithmic scale dependence as predicted by the LO anomalous dimensions of the  $\Delta B = 0$  and  $\Delta B = 1$  operators;
- *IR divergences*: the coefficient functions are infrared finite. We verified that the cancellation of IR divergences also takes place, separately in the full and the effective amplitudes, for the abelian combination of diagrams.

The  $\Delta B = 0$  effective theory is derived from the double insertion of the  $\Delta B = 1$  effective hamiltonian. Therefore, the coefficient functions  $c_k^q$  of the  $\Delta B = 0$  effective theory depend quadratically on the coefficient functions  $C_i$  of the  $\Delta B = 1$  effective hamiltonian, and we can write

$$c_k^q(\mu_0) = \sum_{i,j} C_i(\mu_1) C_j(\mu_1) F_{k,ij}^q(\mu_1, \mu_0). \quad (12)$$

Eq. (12) shows explicitly the scale dependence of the several terms. We denote with  $\mu_1$  the renormalization scale of the  $\Delta B = 1$  effective hamiltonian, whereas  $\mu_0$  is the renormalization scale of the  $\Delta B = 0$  operators. The coefficients  $F_{k,ij}^q$  depend on the renormalization scheme and scale of both the  $\Delta B = 0$  and  $\Delta B = 1$  operators. The dependence on the scale  $\mu_1$  and on the renormalization scheme of the  $\Delta B = 1$  operators actually cancels, order by order in perturbation theory, against the corresponding dependence of the  $\Delta B = 1$  Wilson coefficients  $C_i$ . Therefore, the coefficient functions  $c_k^q$  only depend on the renormalization scheme and scale of the  $\Delta B = 0$  operators. We have chosen to renormalize these operators, in the HQET, in the NDR- $\overline{\text{MS}}$  scheme defined in details in ref. [32].

The four fermion operators, in the effective  $\Delta B = 1$  hamiltonian, are naturally expressed in terms of the weak eigenstates  $d'$  and  $s'$ . For this reason, we can write the coefficients  $F_{k,ij}^q$  of eq. (12) in the form

$$\begin{aligned} F_{k,ij}^d &= \cos^2 \theta_c F_{k,ij}^{d'} + \sin^2 \theta_c F_{k,ij}^{s'} \\ F_{k,ij}^s &= \sin^2 \theta_c F_{k,ij}^{d'} + \cos^2 \theta_c F_{k,ij}^{s'} \end{aligned} \quad (13)$$

for  $i, j = 1, 2$ . In the case  $q = s$ , the coefficient functions also receive contributions from the insertion of the penguin and chromomagnetic operators (diagram  $D_{15}$  of fig. 2). Since the Wilson coefficients  $C_3$ – $C_6$  are small, contributions with a double insertion of penguin operators can be safely neglected. As suggested in [33], a consistent way for implementing this approximation is to consider the coefficients  $C_3$ – $C_6$  as formally of  $\mathcal{O}(\alpha_s)$ . Within this approximation, only single insertions of penguin operators need to be considered at the

	$q = d$			$q = u$			$q = s$		
	LO	NLO ( $m_c = 0$ )	NLO	LO	NLO ( $m_c = 0$ )	NLO	LO	NLO ( $m_c = 0$ )	NLO
$c_1^q$	-0.02	-0.03	-0.03	-0.06	-0.33	-0.29	-0.01	-0.03	-0.03
$c_2^q$	0.02	0.03	0.03	0.00	-0.01	-0.01	0.02	0.03	0.04
$c_3^q$	-0.70	-0.65	-0.67	2.11	2.27	2.34	-0.61	-0.51	-0.56
$c_4^q$	0.79	0.68	0.68	0.00	-0.06	-0.05	0.77	0.61	0.64

Table 1: *Wilson coefficients  $c_k^q$  computed at the LO and NLO, in the latter case with and without the inclusion of charm quark mass corrections at  $\mathcal{O}(\alpha_s)$ . As reference values of the input parameters, we use  $\mu_0 = \mu_1 = m_b = 4.8$  GeV and  $m_c = 1.4$  GeV.*

NLO, and we can write

$$c_k^s = \sum_{i,j=1,2} C_i C_j F_{k,ij}^s + 2 \frac{\alpha_s}{4\pi} C_2 C_{8G} P_{k,28} + 2 \sum_{i=1,2} \sum_{r=3,6} C_i C_r P_{k,ir}, \quad (14)$$

which generalizes eq. (12) for the case  $q = s$ .

The analytical expressions of the coefficients  $F_{k,ij}^q$  and  $P_{k,ij}$  will be given in the appendix. For illustration we present here, in table 1, the numerical values of the coefficients  $c_k^q$  both at LO and NLO and, in the latter case, with and without the inclusion of the  $\mathcal{O}(\alpha_s)$  charm quark mass corrections computed in this paper. As reference values of the input parameters, we use  $\mu_0 = \mu_1 = m_b = 4.8$  GeV and  $m_c = 1.4$  GeV.

By looking at the results shown in table 1, we see that the NLO charm quark mass corrections are rather large for some of the Wilson coefficients, although the total effect of these contributions on the lifetime ratios will be found to be small. The numerical expressions of the lifetime ratios as a function of the  $B$ -parameters will be given in the next section (eq.(26)). In these expressions we will also include an estimate of the theoretical error on the coefficients, coming from the residual NNLO dependence on the renormalization scale  $\mu_1$  and from the uncertainties on the values of the charm and bottom quark masses and the other input parameters.

The combinations  $c_k^u - c_k^d$  ( $k = 1, \dots, 4$ ) of Wilson coefficients, which enter the theoretical expression of the ratio  $\tau(B^+)/\tau(B_d)$ , have been also computed, at the NLO, in ref. [6]. In this case, the corresponding operators are the flavour non-singlet combinations  $\mathcal{O}_k^u - \mathcal{O}_k^d$ , which do not mix, with coefficients proportional to powers of the  $b$  quark mass, with operators of lower dimension. For this reason, the HQE can be also expressed in this case in terms of operators defined in QCD, and this is the choice followed by ref. [6].

In order to compare our results with those of ref. [6], at the NLO, it is necessary to implement the matching, at  $\mathcal{O}(\alpha_s)$ , between QCD and HQET operators. This matching can be written in terms of the Wilson coefficients, in the form

$$\left[ c_k^u(m_b) - c_k^d(m_b) \right]_{\text{QCD}} = \left( 1 + \frac{\alpha_s(m_b)}{4\pi} \hat{S} \right)_{k,l} \left[ c_l^u(m_b) - c_l^d(m_b) \right]_{\text{HQET}}, \quad (15)$$

where a common renormalization scale  $\mu = m_b$  has been chosen for all the coefficients. The matrix  $\hat{S}$  has been computed in ref. [5]. It depends on the renormalization schemes of both QCD and HQET operators. In this paper, we have chosen to renormalize the HQET operators in the NDR- $\overline{\text{MS}}$  scheme of ref. [32]. By choosing for the QCD operators the NDR- $\overline{\text{MS}}$  scheme of ref. [6], defined in ref. [34], one finds that the matrix  $\hat{S}$  is given by <sup>3</sup>

$$\hat{S} = \begin{pmatrix} 32/3 & 0 & 22/9 & 1/3 \\ -16/3 & -16/3 & 4/9 & -2/9 \\ 11 & 3/2 & -7/2 & 1/4 \\ 2 & -1 & 3 & 5/2 \end{pmatrix} \quad (16)$$

By using eq. (15), we have verified that our results for the combinations  $c_k^u - c_k^d$  agree with those of ref. [6]. Note that in the notation of [6] the labels  $u$  and  $d$  are interchanged with respect to our convention and that the Wilson coefficients are defined with a relative factor of 3. The numerical comparison of our results with those shown in table 1 of ref. [6], however, shows some differences, particularly at the LO. The reason is that some of the coefficient functions, because of large cancellations, are extremely sensitive to the value of the coupling constant  $\alpha_s(\mu)$ .

## 4 Theoretical estimates of the lifetime ratios

In this section we present the theoretical estimates of the lifetime ratios of beauty hadrons, obtained by using the NLO expressions of the Wilson coefficients and the lattice determinations of the relevant hadronic matrix elements [7]-[10].

The HQE for the ratio of inclusive widths of beauty hadrons is expressed by eq. (9), up to and including  $1/m_b^3$  spectator effects. The combinations of hadronic parameters entering this formula at order  $1/m_b^2$  can be evaluated from the heavy hadron spectroscopy [35], and one obtains the estimate

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.00 - \Delta_{\text{spec}}^{B^+}, \quad \frac{\tau(B_s)}{\tau(B_d)} = 1.00 - \Delta_{\text{spec}}^{B_s}, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.98(1) - \Delta_{\text{spec}}^{\Lambda}, \quad (17)$$

where the  $\Delta$ s represent the  $1/m_b^3$  contributions of hard spectator effects

$$\Delta_{\text{spec}}^{H_b} = \frac{96\pi^2}{m_b^3 c(3)} \sum_k c_k^{(6)} \left( \frac{\langle O_k^{(6)} \rangle_{H_b}}{2M_{H_b}} - \frac{\langle O_k^{(6)} \rangle_{B_d}}{2M_{B_d}} \right). \quad (18)$$

---

<sup>3</sup>This matrix differs from the matrix  $\hat{C}_1$  given in ref. [5] by a linear transformation, since a basis of operators different from eq. (10) has been chosen in that paper. More specifically,  $\hat{S} = -M\hat{C}_1^T M^{-1}$ , where  $M$  is defined in ref. [5].

Note that, by neglecting these contributions, the  $1/m_b^2$  predictions of eq. (17) are incompatible with the experimental results of eq. (2).

In parametrizing the matrix elements of the dimension-six current-current operators, we follow the analysis of ref. [5] and we distinguish two cases, depending on whether or not the light quark  $q$  of the operator enters as a valence quark in the external hadronic state. Correspondingly, we introduce different  $B$ -parameters for the valence and non-valence contributions. For the  $B$ -meson matrix elements, we write the matrix elements of the non-valence operators in the form

$$\frac{\langle B_q | \mathcal{O}_k^{q'} | B_q \rangle}{2M_{B_q}} = \frac{f_{B_q}^2 M_{B_q}}{2} \delta_k^{q'q} \quad \text{for } q \neq q', \quad (19)$$

while, in the case of the valence contributions ( $q = q'$ ), we write

$$\begin{aligned} \frac{\langle B_q | \mathcal{O}_1^q | B_q \rangle}{2M_{B_q}} &= \frac{f_{B_q}^2 M_{B_q}}{2} (B_1^q + \delta_1^{qq}), & \frac{\langle B_q | \mathcal{O}_3^q | B_q \rangle}{2M_{B_q}} &= \frac{f_{B_q}^2 M_{B_q}}{2} (\varepsilon_1^q + \delta_3^{qq}), \\ \frac{\langle B_q | \mathcal{O}_2^q | B_q \rangle}{2M_{B_q}} &= \frac{f_{B_q}^2 M_{B_q}}{2} (B_2^q + \delta_2^{qq}), & \frac{\langle B_q | \mathcal{O}_4^q | B_q \rangle}{2M_{B_q}} &= \frac{f_{B_q}^2 M_{B_q}}{2} (\varepsilon_2^q + \delta_4^{qq}). \end{aligned} \quad (20)$$

The parameters  $\delta_k^{qq}$  in eq. (20) are defined as the  $\delta_k^{qq'}$  of eq. (19) in the limit of degenerate quark masses ( $m_q = m_{q'}$ ). In the VIA,  $B_1^q = B_2^q = 1$  while the  $\varepsilon$  parameters and all the  $\delta$ s vanish. Note that, in the  $SU(2)$  limit, the parameters  $B_{1,2}^d$  and  $\varepsilon_{1,2}^d$  express the matrix elements of the non-singlet operator  $\mathcal{O}_k^u - \mathcal{O}_k^d$  between external  $B$ -meson states.

The reason to distinguish between valence and non valence contributions, is that only the former have been computed so far by using lattice QCD simulations [7, 10]. A non-perturbative lattice calculation of the  $\delta$  parameters would be also possible, in principle. However, it requires to deal with the difficult problem of subtractions of power-divergences, which has prevented so far the calculation of the corresponding diagrams.

To complete the definitions of the  $B$ -parameters for the  $B$ -mesons, we introduce a parameter for the matrix element of the penguin operator

$$\frac{\langle B_q | \mathcal{O}_P | B_q \rangle}{2M_{B_q}} = \frac{f_B^2 M_B}{2} P^q. \quad (21)$$

We now define the  $B$ -parameters for the  $\Lambda_b$  baryon. Up to  $1/m_b$  corrections, the matrix elements of the operators  $\mathcal{O}_2^q$  and  $\mathcal{O}_4^q$ , between external  $\Lambda_b$  states, can be related to the matrix elements of the operators  $\mathcal{O}_1^q$  and  $\mathcal{O}_3^q$  [4]

$$\langle \Lambda_b | \mathcal{O}_1^q | \Lambda_b \rangle = -2 \langle \Lambda_b | \mathcal{O}_2^q | \Lambda_b \rangle, \quad \langle \Lambda_b | \mathcal{O}_3^q | \Lambda_b \rangle = -2 \langle \Lambda_b | \mathcal{O}_4^q | \Lambda_b \rangle. \quad (22)$$

For the independent matrix elements, assuming  $SU(2)$  symmetry, we define

$$\begin{aligned} \frac{\langle \Lambda_b | \mathcal{O}_1^q | \Lambda_b \rangle}{2M_{\Lambda_b}} &= \frac{f_B^2 M_B}{2} (L_1 + \delta_1^{\Lambda q}) \quad \text{for } q = u, d, \\ \frac{\langle \Lambda_b | \mathcal{O}_3^q | \Lambda_b \rangle}{2M_{\Lambda_b}} &= \frac{f_B^2 M_B}{2} (L_2 + \delta_2^{\Lambda q}) \quad \text{for } q = u, d, \end{aligned}$$

$$\begin{aligned}
\frac{\langle \Lambda_b | \mathcal{O}_1^q | \Lambda_b \rangle}{2M_{\Lambda_b}} &= \frac{f_B^2 M_B}{2} \delta_1^{\Lambda q} \quad \text{for } q = s, c, \\
\frac{\langle \Lambda_b | \mathcal{O}_3^q | \Lambda_b \rangle}{2M_{\Lambda_b}} &= \frac{f_B^2 M_B}{2} \delta_2^{\Lambda q} \quad \text{for } q = s, c, \\
\frac{\langle \Lambda_b | \mathcal{O}_P | \Lambda_b \rangle}{2M_{\Lambda_b}} &= \frac{f_B^2 M_B}{2} P^\Lambda.
\end{aligned} \tag{23}$$

In analogy with the  $B$ -meson case, the parameters  $L_1$  and  $L_2$  represent the valence contributions computed so far by current lattice calculations [8, 9].

In terms of valence and non valence  $B$ -parameters, the quantities  $\Delta_{\text{spec}}^{H_b}$  of eq. (18), which represent the spectator contributions to the lifetime ratios, are expressed in the form

$$\begin{aligned}
\Delta_{\text{spec}}^{B^+} &= 48\pi^2 \frac{f_B^2 M_B}{m_b^3 c^{(3)}} \sum_{k=1}^4 (c_k^u - c_k^d) \mathcal{B}_k^d, \\
\Delta_{\text{spec}}^{B_s} &= 48\pi^2 \frac{f_B^2 M_B}{m_b^3 c^{(3)}} \left\{ \sum_{k=1}^4 \left[ r c_k^s \mathcal{B}_k^s - c_k^d \mathcal{B}_k^d + (c_k^u + c_k^d) (r \delta_k^{ds} - \delta_k^{dd}) + \right. \right. \\
&\quad \left. \left. c_k^s (r \delta_k^{ss} - \delta_k^{sd}) + c_k^c (r \delta_k^{cs} - \delta_k^{cd}) \right] + c_P (r P^s - P^d) \right\}, \tag{24} \\
\Delta_{\text{spec}}^\Lambda &= 48\pi^2 \frac{f_B^2 M_B}{m_b^3 c^{(3)}} \left\{ \sum_{k=1}^4 \left[ (c_k^u + c_k^d) \mathcal{L}_k^\Lambda - c_k^d \mathcal{B}_k^d + (c_k^u + c_k^d) (\delta_k^{\Lambda d} - \delta_k^{dd}) + \right. \right. \\
&\quad \left. \left. c_k^s (\delta_k^{\Lambda s} - \delta_k^{sd}) + c_k^c (\delta_k^{\Lambda c} - \delta_k^{cd}) \right] + c_P (P^\Lambda - P^d) \right\}.
\end{aligned}$$

where  $r$  denotes the ratio  $(f_{B_s}^2 M_{B_s})/(f_B^2 M_B)$  and, in order to simplify the notation, we have defined the vectors of parameters

$$\begin{aligned}
\vec{\mathcal{B}}^q &= \{B_1^q, B_2^q, \varepsilon_1^q, \varepsilon_1^q\}, \\
\vec{\mathcal{L}} &= \{L_1, -L_1/2, L_2, -L_2/2\}, \\
\vec{\delta}^{\Lambda q} &= \{\delta_1^{\Lambda q}, -\delta_1^{\Lambda q}/2, \delta_2^{\Lambda q}, -\delta_2^{\Lambda q}/2\}.
\end{aligned} \tag{25}$$

An important consequence of eq. (24) is that, because of the  $SU(2)$  symmetry, the non-valence ( $\delta$ s) and penguin ( $P$ s) contributions cancel out in the expressions of the lifetime ratio  $\tau(B^+)/\tau(B_d)$ . Thus, the theoretical prediction of this ratio is at present the most accurate, since it depends only on the non-perturbative parameters actually computed by current lattice calculations. The prediction of the ratio  $\tau(\Lambda_b)/\tau(B_d)$ , instead, is affected by both the uncertainties on the values of the  $\delta$  and  $P$  parameters, and by the unknown expressions of the Wilson coefficients  $c_k^c$  and  $c_P$  at the NLO. For the ratio  $\tau(B_s)/\tau(B_d)$  the same uncertainties exist, although their effect is expected to be smaller, since the contributions of non-valence and penguin operators cancel, in this case, in the limit of exact  $SU(3)$  symmetry.

In the numerical analysis of the ratios  $\tau(B_s)/\tau(B_d)$  and  $\tau(\Lambda_b)/\tau(B_d)$ , we will neglect the non-valence and penguin contributions. The non-valence contributions vanish in the VIA, and present phenomenological estimates indicate that the corresponding matrix elements are suppressed, with respect to the valence contributions, by at least one order

of magnitude [36, 37]. On the other hand, the matrix elements of the penguin operators are not expected to be smaller than those of the valence operators. Since the coefficient function  $c_P$  vanishes at the LO, this contribution is expected to have the size of a typical NLO corrections. Thus, from a theoretical point of view, a quantitative evaluation of the non-valence and penguin operator matrix elements would be of the greatest interest to improve the determination of the  $\Lambda_B$  lifetime.

By neglecting the non valence and penguin contributions, we obtain from eq. (24) the following numerical expressions for the spectator effects

$$\begin{aligned}
\Delta_{\text{spec}}^{B^+} &= -0.06(2) B_1^d - 0.010(3) B_2^d + 0.7(2) \varepsilon_1^d - 0.18(5) \varepsilon_2^d, \\
\Delta_{\text{spec}}^{B_s} &= -0.010(2) B_1^s + 0.011(3) B_2^s - 0.16(4) \varepsilon_1^s + 0.18(5) \varepsilon_2^s \\
&\quad + 0.008(2) B_1^d - 0.008(2) B_2^d + 0.16(4) \varepsilon_1^d - 0.16(4) \varepsilon_2^d, \\
\Delta_{\text{spec}}^\Lambda &= -0.08(2) L_1 + 0.33(8) L_2 \\
&\quad + 0.008(2) B_1^d - 0.008(2) B_2^d + 0.16(4) \varepsilon_1^d - 0.16(4) \varepsilon_2^d,
\end{aligned} \tag{26}$$

These formulae, which are accurate at the NLO, represent the main result of this paper. The errors on the coefficients take into account both the residual NNLO dependence on the renormalization scale of the  $\Delta B = 1$  operators and the theoretical uncertainties on the input parameters. To estimate the former, the scale  $\mu_1$  has been varied in the interval between  $m_b/2$  and  $2m_b$ . For the charm and bottom quark masses, and the B meson decay constants we have used the central values and errors given in table 2. The strong coupling constant has been fixed at the value  $\alpha_s(m_Z) = 0.118$ . The parameter  $c^{(3)}$  in eq. (24) is a function of the ratio  $m_c^2/m_b^2$ , and such a dependence has been consistently taken into account in the numerical analysis and in the estimates of the errors. For the range of masses given in table 2,  $c^{(3)}$  varies in the interval  $c^{(3)} = 3.4 \div 4.2$  [22]-[24].

As discussed in the previous section, the HQE for the ratio  $\tau(B^+)/\tau(B_d)$  can be also expressed in terms of operators defined in QCD. The corresponding coefficient functions can be evaluated by applying the matching defined in eq. (15). In this way, we obtain the expression

$$\Delta_{\text{spec}}^{B^+} = -0.05(1) \bar{B}_1^d - 0.007(2) \bar{B}_2^d + 0.7(2) \bar{\varepsilon}_1^d - 0.15(4) \bar{\varepsilon}_2^d \tag{27}$$

where the  $\bar{B}$  and  $\bar{\varepsilon}$  parameters are now defined in terms of matrix elements of QCD operators. This expression is in agreement with the result obtained in ref. [6].

The errors quoted on the coefficients in eq. (26) are strongly correlated, since they originate from the theoretical uncertainties on the same set of input parameters. For this reason, in order to evaluate the lifetime ratios, we have not used directly eq. (26). Instead, we have performed a bayesian statistical analysis by implementing a short Monte Carlo calculation. The input parameters have been extracted with flat distributions, assuming as central values and standard deviations the values given in table 2. The results for the  $B$ -parameters are based on the lattice determinations of refs. [7]-[10].<sup>4</sup> As discussed in details in ref. [5], we have included in the errors an estimate of the uncertainties not taken

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<sup>4</sup>For recent estimates of these matrix elements based on QCD sum rules, see refs. [38]-[41].

$B_1^d = 1.2 \pm 0.2$	$B_1^s = 1.0 \pm 0.2$
$B_2^d = 0.9 \pm 0.1$	$B_2^s = 0.8 \pm 0.1$
$\varepsilon_1^d = 0.04 \pm 0.01$	$\varepsilon_1^s = 0.03 \pm 0.01$
$\varepsilon_2^d = 0.04 \pm 0.01$	$\varepsilon_2^s = 0.03 \pm 0.01$
$L_1 = -0.2 \pm 0.1$	$L_2 = 0.2 \pm 0.1$
$m_b = 4.8 \pm 0.1 \text{ GeV} \quad m_b - m_c = 3.40 \pm 0.06 \text{ GeV}$	
$f_B = 200 \pm 25 \text{ MeV} \quad f_{B_s}/f_B = 1.16 \pm 0.04$	

Table 2: *Central values and standard deviations of the input parameters used in the numerical analysis. The values of  $m_b$  and  $m_c$  refer to the pole mass definitions of these quantities.*

into account in the original papers. The QCD results for the  $B$  meson  $B$ -parameters of ref. [10] have been converted to HQET by using eq. (15).<sup>5</sup> The contributions of all the  $\delta$  and  $P$  parameters have been neglected.

In this way we obtain the NLO predictions for the lifetimes ratios which have been also quoted in the introduction

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.06 \pm 0.02, \quad \frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.90 \pm 0.05. \quad (28)$$

The central values and errors correspond to the average and the standard deviation of the theoretical distributions. These distributions are shown in fig. 4, together with the experimental ones. We mention that uncertainties coming from the residual scale dependence in the results of eq. (28) represent less than 20% of the quoted errors.

In conclusion we find that, with the inclusion of the NLO corrections, the theoretical prediction for the ratio  $\tau(B^+)/\tau(B_d)$  turns out to be in very good agreement with the experimental measurement, given in eq. (2). For the ratios  $\tau(B_s)/\tau(B_d)$  and  $\tau(\Lambda_b)/\tau(B_d)$  the agreement is also very satisfactory, and the difference between theoretical and experimental determinations is at the  $1\sigma$  level. We have pointed out, however, that the theoretical predictions are less accurate in these cases, since a reliable estimate of the contribution of the non-valence and penguin operators cannot be performed yet. We also found that the NLO charm quark mass corrections computed in this paper are rather large for some of the Wilson coefficients. Nevertheless, the total effect of these contributions on the lifetime ratios is numerically small.

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<sup>5</sup>With respect to ref. [5], we use for the  $B$ -meson  $B$ -parameters the results updated in ref. [10].

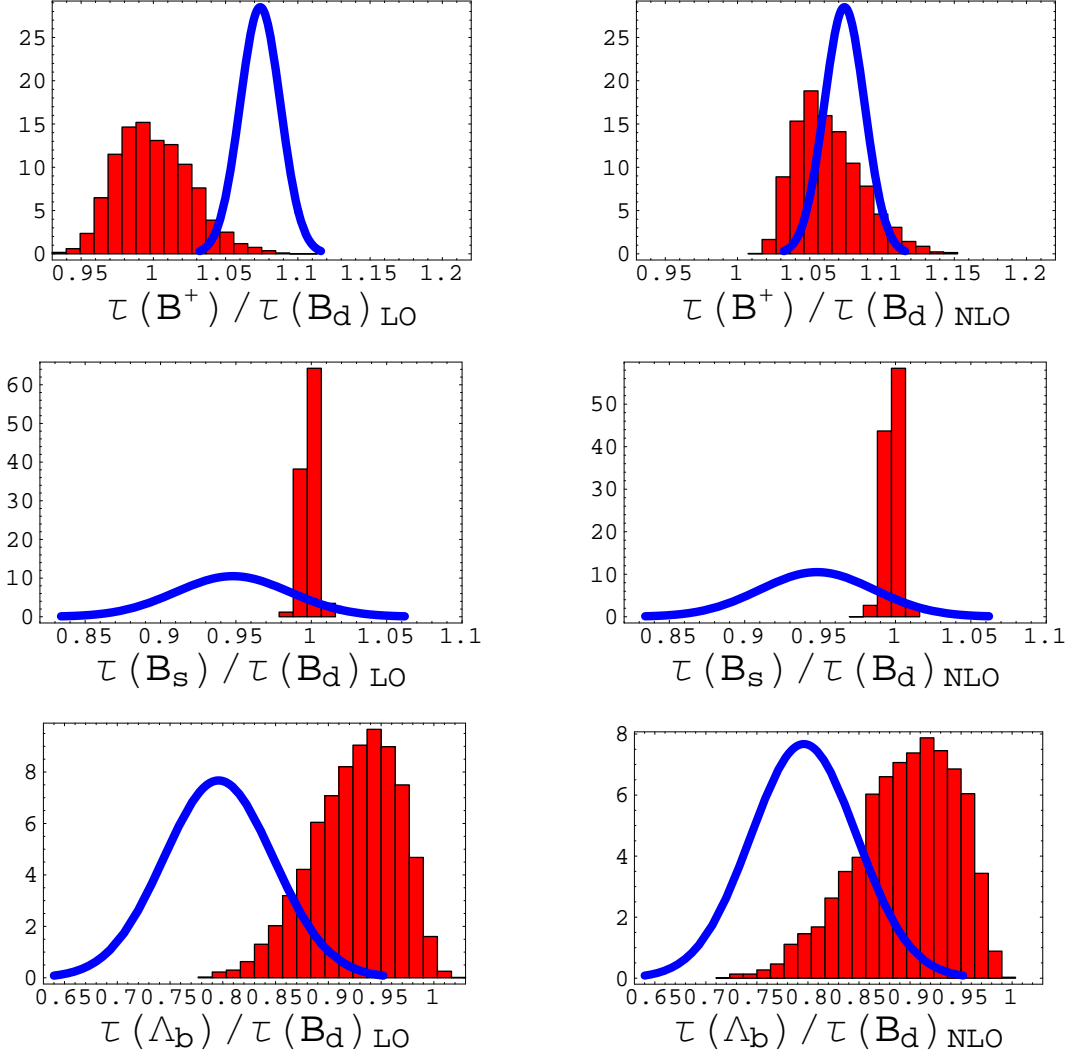


Figure 4: *Theoretical (histogram) vs experimental (solid line) distributions of lifetime ratios. The theoretical predictions are shown at the LO (left) and NLO (right).*



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## Appendix

In this appendix we collect the analytical expressions of the Wilson coefficients, both at the LO and NLO. The LO coefficients have been computed in refs. [4, 25] and are reported here for completeness.

We distinguish the leading and next-to-leading contributions in the coefficients  $F_{k,ij}^q$  by writing the expansion

$$F_{k,ij}^q = A_{k,ij}^q + \frac{\alpha_s}{4\pi} B_{k,ij}^q, \quad (29)$$

( $q = u, d', s'$ ). Since, by definition, the coefficients  $A_{k,ij}^q$  and  $B_{k,ij}^q$  are symmetric in the indices  $i$  and  $j$ , we will only present results for  $i \leq j$ .

The LO coefficients  $A_{k,ij}^q$  read

$$\begin{aligned} A_{1,11}^u &= \frac{(1-z)^2}{3}, & A_{1,12}^u &= (1-z)^2, & A_{1,22}^u &= \frac{(1-z)^2}{3}, \\ A_{2,11}^u &= 0, & A_{2,12}^u &= 0, & A_{2,22}^u &= 0, \\ A_{3,11}^u &= 2(1-z)^2, & A_{3,12}^u &= 0, & A_{3,22}^u &= 2(1-z)^2, \\ A_{4,11}^u &= 0, & A_{4,12}^u &= 0, & A_{4,22}^u &= 0. \end{aligned} \quad (30)$$

$$\begin{aligned} A_{1,11}^{d'} &= \frac{-(1-z)^2(2+z)}{2}, & A_{1,12}^{d'} &= \frac{-(1-z)^2(2+z)}{6}, & A_{1,22}^{d'} &= \frac{-(1-z)^2(2+z)}{18}, \\ A_{2,11}^{d'} &= (1-z)^2(1+2z), & A_{2,12}^{d'} &= \frac{(1-z)^2(1+2z)}{3}, & A_{2,22}^{d'} &= \frac{(1-z)^2(1+2z)}{9}, \\ A_{3,11}^{d'} &= 0, & A_{3,12}^{d'} &= 0, & A_{3,22}^{d'} &= \frac{-(1-z)^2(2+z)}{3}, \\ A_{4,11}^{d'} &= 0, & A_{4,12}^{d'} &= 0, & A_{4,22}^{d'} &= \frac{2(1-z)^2(1+2z)}{3}. \end{aligned} \quad (31)$$

$$\begin{aligned}
A_{1,11}^{s'} &= -\sqrt{1-4z} (1-z), & A_{1,12}^{s'} &= \frac{-\sqrt{1-4z} (1-z)}{3}, & A_{1,22}^{s'} &= \frac{-\sqrt{1-4z} (1-z)}{9}, \\
A_{2,11}^{s'} &= \sqrt{1-4z} (1+2z), & A_{2,12}^{s'} &= \frac{\sqrt{1-4z} (1+2z)}{3}, & A_{2,22}^{s'} &= \frac{\sqrt{1-4z} (1+2z)}{9}, \\
A_{3,11}^{s'} &= 0, & A_{3,12}^{s'} &= 0, & A_{3,22}^{s'} &= \frac{-2\sqrt{1-4z} (1-z)}{3}, \\
A_{4,11}^{s'} &= 0, & A_{4,12}^{s'} &= 0, & A_{4,22}^{s'} &= \frac{2\sqrt{1-4z} (1+2z)}{3}.
\end{aligned} \tag{32}$$

where  $z = m_c^2/m_b^2$ .

The NLO results for the coefficients  $B_{k,ij}^q$  have been obtained in the NDR- $\overline{\text{MS}}$  scheme of ref. [34] for the  $\Delta B = 1$  operators and the NDR- $\overline{\text{MS}}$  scheme of ref. [32] for the  $\Delta B = 0$ , HQET operators. We find

$$\begin{aligned}
B_{1,11}^u &= \frac{-8(1-z)(164+12\pi^2-109z+5z^2)}{81} - \frac{16(1-z)^2(6\log x_1 + (1+z)\log(1-z))}{9} - \\
&\quad \frac{16z(12+z-3z^2)\log z}{27} - \frac{16(1-3z)(1-z)(\log(1-z)\log z + 2\text{Li}_2(z))}{9}, \\
B_{2,11}^u &= \frac{16(1-z)(4-11z+10z^2)}{81} + \frac{16z^2\log z}{27}, \\
B_{3,11}^u &= \frac{((1-z)(751-12\pi^2(7-9z)-1199z-26z^2))}{27} + \\
&\quad 8\log x_1(1-z)^2 - \frac{4(26-z)(1-z)^2\log(1-z)}{3} - \\
&\quad \frac{2z(192-119z+6z^2)\log z}{9} - \frac{8(4-3z)(1-z)(\log(1-z)\log z + 2\text{Li}_2(z))}{3}, \\
B_{4,11}^u &= \frac{-8(1-z)(11-10z-13z^2)}{27} + \frac{32z^2\log z}{9},
\end{aligned} \tag{33}$$

$$\begin{aligned}
B_{1,12}^u &= \frac{-8(1-z)(13+2\pi^2-2z+z^2)}{9} - \frac{4(1-z)^2(6\log x_0 + 4\log(1-z))}{3} - \\
&\quad \frac{16(3-z)z\log z}{3} + \frac{16(1-z)z(\log(1-z)\log z + 2\text{Li}_2(z))}{3}, \\
B_{2,12}^u &= \frac{32(1-z)^3}{9}, \\
B_{3,12}^u &= \frac{-2(1-z)(125+6\pi^2-103z+2z^2)}{9} + \\
&\quad (1-z)^2(6\log x_0 - 24\log x_1 - 4z\log(1-z)) - \frac{4z(3+4z-3z^2)\log z}{3} - \\
&\quad 4(1-2z)(1-z)(\log(1-z)\log z + 2\text{Li}_2(z)), \\
B_{4,12}^u &= \frac{-4(1-z)(2-z-4z^2)}{9} + \frac{4z^2\log z}{3},
\end{aligned} \tag{34}$$

$$B_{1,22}^u = \frac{-8(1-z)(164+12\pi^2-109z+5z^2)}{81} -$$

$$\begin{aligned}
& \frac{16(1-z)^2(6\log x_1 + (1+z)\log(1-z))}{9} - \frac{16z(12+z-3z^2)\log z}{27} - \\
& \frac{16(1-3z)(1-z)(\log(1-z)\log z + 2\text{Li}_2(z))}{9}, \\
B_{2,22}^u &= \frac{16(1-z)(4-11z+10z^2)}{81} + \frac{16z^2\log z}{27}, \\
B_{3,22}^u &= \frac{(1-z)(751-875z-26z^2-12\pi^2(7+9z))}{27} + \\
& \frac{4(1-z)^2(6\log x_1 - (17-z)\log(1-z))}{3} - \frac{2(28-3z)(3-2z)z\log z}{9} + \\
& \frac{8(1-z)(5+3z)(\log(1-z)\log z + 2\text{Li}_2(z))}{3}, \\
B_{4,22}^u &= \frac{-8(1-z)(11-10z-13z^2)}{27} + \frac{32z^2\log z}{9}, \tag{35}
\end{aligned}$$

$$\begin{aligned}
B_{1,11}^{d'} &= \frac{2(1-z)(2-z)(5+3z)}{3} + \frac{4(1-z)^2(4+5z)\log(1-z)}{3} + \\
& \frac{4z(5+4z-5z^2)\log z}{3} + \\
& \frac{2(1-z)^2(2+z)(6\log x_0 - 4(\log(1-z)\log z + 2\text{Li}_2(z)))}{3}, \\
B_{2,11}^{d'} &= \frac{-4(1-z)(5-7z+6z^2)}{3} - \frac{8(1-z)^2(2+10z-3z^2)\log(1-z)}{3} + \\
& \frac{8z(2-17z+16z^2-3z^3)\log z}{3} - \\
& \frac{4(1-z)^2(1+2z)(6\log x_0 - 4(\log(1-z)\log z + 2\text{Li}_2(z)))}{3}, \\
B_{3,11}^{d'} &= \frac{-((1-z)(205+7z-110z^2))}{18} - \\
& \frac{3(1-z)^2(2+z)(2\log x_0 - 2\log(1-z))}{2} - \frac{z^2(6+11z)\log z}{3}, \\
B_{4,11}^{d'} &= \frac{(1-z)(95+104z-211z^2)}{9} + \\
& (1-z)^2(1+2z)(6\log x_0 - 6\log(1-z)) - \frac{4(12-11z)z^2\log z}{3}, \tag{36}
\end{aligned}$$

$$\begin{aligned}
B_{1,12}^{d'} &= \frac{8\pi^2(1-z)z(2+z)}{27} + \frac{2(1-z)(42-15z-25z^2)}{9} + \\
& \frac{2(1-z)^2(2+z)(2\log x_0 + 4\log x_1)}{3} + \\
& \frac{4(1-z)^2(2+z+3z^2)\log(1-z)}{9z} + \frac{4z(1+6z-6z^2)\log z}{9} - \\
& \frac{8(1-z)(2-z)(2+z)(\log(1-z)\log z + 2\text{Li}_2(z))}{9}, \\
B_{2,12}^{d'} &= \frac{-16\pi^2(1-z)z(1+2z)}{27} - \frac{4(1-z)(21+27z-38z^2)}{9} - \\
& \frac{4(1-z)^2(1+2z)(2\log x_0 + 4\log x_1)}{3} -
\end{aligned}$$

$$\begin{aligned}
& \frac{8(1-z)^2(1+2z+6z^2-3z^3)\log(1-z)}{9z} + \\
& \frac{8z(4-24z+18z^2-3z^3)\log z}{9} + \\
& \frac{16(1-z)(2-z)(1+2z)(\log(1-z)\log z+2\text{Li}_2(z))}{9}, \\
B_{3,12}^{d'} = & \frac{2\pi^2(1-z)z(2+z)}{9} + \frac{(1-z)(83-151z-88z^2)}{54} - \\
& \frac{(1-z)^2(2+z)(2\log x_0-4\log x_1)}{2} + \\
& \frac{(1-z)^2(1+z)(2+z)\log(1-z)}{3z} - \frac{2z(6+7z^2)\log z}{9} - \\
& \frac{2(1-z)(2+z)(\log(1-z)\log z+2\text{Li}_2(z))}{3}, \\
B_{4,12}^{d'} = & \frac{-4\pi^2(1-z)z(1+2z)}{9} - \frac{(1-z)(49+202z-185z^2)}{27} + \\
& (1-z)^2(1+2z)(2\log x_0-4\log x_1) - \\
& \frac{2(1-z)^2(1+z)(1+2z)\log(1-z)}{3z} + \frac{2z(6-45z+28z^2)\log z}{9} + \\
& \frac{4(1-z)(1+2z)(\log(1-z)\log z+2\text{Li}_2(z))}{3}, \tag{37}
\end{aligned}$$

$$\begin{aligned}
B_{1,22}^{d'} = & \frac{16\pi^2(1-z)z(2+z)}{81} + \frac{2(1-z)(461-286z-313z^2)}{243} + \\
& \frac{16(1-z)^2(2+z)\log x_1}{9} + \frac{16(1-z)^2(1+z+z^2)\log(1-z)}{27z} - \\
& \frac{4z(9-18z+32z^2)\log z}{81} - \\
& \frac{8(1-z)(3-z)(2+z)(\log(1-z)\log z+2\text{Li}_2(z))}{27}, \\
B_{2,22}^{d'} = & \frac{-32\pi^2(1-z)z(1+2z)}{81} - \frac{4(1-z)(238+445z-527z^2)}{243} - \\
& \frac{32(1-z)^2(1+2z)\log x_1}{9} - \\
& \frac{8(1-z)^2(2+5z+8z^2-3z^3)\log(1-z)}{27z} + \\
& \frac{8z(18-117z+82z^2-9z^3)\log z}{81} + \\
& \frac{16(1-z)(3-z)(1+2z)(\log(1-z)\log z+2\text{Li}_2(z))}{27}, \\
B_{3,22}^{d'} = & \frac{2\pi^2(1-z)(2+z)(9+7z)}{27} - \frac{(1-z)(937-449z-746z^2)}{81} - \\
& \frac{4(1-z)^2(2+z)\log x_1}{3} - \frac{4(1-z)^2(1-17z-8z^2)\log(1-z)}{9z} + \\
& \frac{2z(72-9z-20z^2)\log z}{27} - \\
& \frac{4(1-z)(2+z)(3+5z)(\log(1-z)\log z+2\text{Li}_2(z))}{9},
\end{aligned}$$

$$\begin{aligned}
B_{4,22}^{d'} = & \frac{-4\pi^2(1-z)(1+2z)(9+7z)}{27} + \frac{(1-z)(715+1003z-2804z^2)}{81} + \\
& \frac{8(1-z)^2(1+2z)\log x_1}{3} + \\
& \frac{2(1-z)^2(2-31z-64z^2-3z^3)\log(1-z)}{9z} + \\
& \frac{2z(36-288z+62z^2+9z^3)\log z}{27} + \\
& \frac{8(1-z)(1+2z)(3+5z)(\log(1-z)\log z+2\text{Li}_2(z))}{9}, \tag{38}
\end{aligned}$$

$$\begin{aligned}
B_{1,11}^{s'} = & \frac{4\sqrt{1-4z}(1-z)(5+6z)}{3} + \frac{8(6-13z-2z^2+6z^3)\log\sigma}{3} + \\
& \frac{4\sqrt{1-4z}(1-z)(6\log x_0+8\log(1-4z)-12\log z)}{3} - \\
& \frac{16}{3}(1-2z)(1-z)(3\log^2\sigma+2\log\sigma\log(1-4z)-3\log\sigma\log z+4\text{Li}_2(\sigma)+ \\
& 2\text{Li}_2(\sigma^2)), \\
B_{2,11}^{s'} = & -\frac{4\sqrt{1-4z}(1-4z)(5+6z)}{3} - \frac{16(3-2z-7z^2+12z^3)\log\sigma}{3} + \\
& 16(1-2z)(1+2z)\log^2\sigma + \frac{32(1-2z)(1+2z)\log\sigma\log(1-4z)}{3} - \\
& \frac{4\sqrt{1-4z}(1+2z)(6\log x_0+8\log(1-4z)-12\log z)}{3} - \\
& 16(1-2z)(1+2z)\log\sigma\log z + \frac{64(1-2z)(1+2z)\text{Li}_2(\sigma)}{3} + \\
& \frac{32(1-2z)(1+2z)\text{Li}_2(\sigma^2)}{3}, \\
B_{3,11}^{s'} = & \frac{-(\sqrt{1-4z}(205+14z+24z^2))}{18} + \frac{2(9-27z-6z^2+4z^3)\log\sigma}{3} - \\
& 3\sqrt{1-4z}(1-z)(2\log x_0-2\log(1-4z)+2\log z), \\
B_{4,11}^{s'} = & \frac{\sqrt{1-4z}(95+208z+48z^2)}{9} - \frac{2(9-6z^2+16z^3)\log\sigma}{3} + \\
& 3\sqrt{1-4z}(1+2z)(2\log x_0-2\log(1-4z)+2\log z), \tag{39}
\end{aligned}$$

$$\begin{aligned}
B_{1,12}^{s'} = & \frac{\sqrt{1-4z}(26-17z-6z^2)}{3} - \frac{(1-60z+146z^2-36z^4)\log\sigma}{9z} + \\
& \frac{4\sqrt{1-4z}(1-z)(6\log x_0+12\log x_1+8\log(1-4z))}{9} + \\
& \frac{\sqrt{1-4z}(1-58z+72z^2)\log z}{9z} - \\
& \frac{16}{9}(1-2z)(1-z)(3\log^2\sigma+2\log\sigma\log(1-4z)-3\log\sigma\log z+4\text{Li}_2(\sigma)+ \\
& 2\text{Li}_2(\sigma^2)), \\
B_{2,12}^{s'} = & \frac{-4\sqrt{1-4z}(7-3z)(1+2z)}{3} + \frac{4(1-15z-4z^2+48z^3-36z^4)\log\sigma}{9z} - \\
& \frac{4\sqrt{1-4z}(1+2z)(6\log x_0+12\log x_1+8\log(1-4z))}{9} -
\end{aligned}$$

$$\begin{aligned}
& \frac{4\sqrt{1-4z}(1-13z-36z^2)\log z}{9z} + \\
& \frac{16}{9}(1-2z)(1+2z)(3\log^2\sigma + 2\log\sigma\log(1-4z) - 3\log\sigma\log z + 4\text{Li}_2(\sigma) + \\
& 2\text{Li}_2(\sigma^2)) , \\
B_{3,12}^{s'} &= \frac{\sqrt{1-4z}(112-523z+6z^2)}{108} - \frac{(3-108z+342z^2+4z^4)\log\sigma}{36z} - \\
& \frac{\sqrt{1-4z}(1-z)(2\log x_0 - 4\log x_1 - 2\log(1-4z))}{12z} + \\
& \frac{\sqrt{1-4z}(1-34z+48z^2)\log z}{12z} , \\
B_{4,12}^{s'} &= \frac{-(\sqrt{1-4z}(49+215z+6z^2))}{27} + \frac{(3-27z-36z^2+72z^3+4z^4)\log\sigma}{9z} + \\
& \frac{\sqrt{1-4z}(1+2z)(2\log x_0 - 4\log x_1 - 2\log(1-4z))}{3z} - \\
& \frac{\sqrt{1-4z}(1-7z-24z^2)\log z}{3z} , \tag{40} \\
\\
B_{1,22}^{s'} &= \frac{2\sqrt{1-4z}(407-491z-78z^2)}{243} - \frac{2(3-144z+390z^2-52z^4)\log\sigma}{81z} + \\
& \frac{8\sqrt{1-4z}(1-z)(12\log x_1 + 7\log(1-4z))}{27} + \frac{2\sqrt{1-4z}(1-46z+60z^2)\log z}{27z} - \\
& \frac{16}{27}(1-2z)(1-z)(3\log^2\sigma + 2\log\sigma\log(1-4z) - 3\log\sigma\log z + 4\text{Li}_2(\sigma) + \\
& 2\text{Li}_2(\sigma^2)) , \\
B_{2,22}^{s'} &= \frac{-8\sqrt{1-4z}(119+256z-78z^2)}{243} + \frac{8(3-36z-24z^2+108z^3-52z^4)\log\sigma}{81z} - \\
& \frac{8\sqrt{1-4z}(1+2z)(12\log x_1 + 7\log(1-4z))}{27} - \frac{8\sqrt{1-4z}(1-10z-30z^2)\log z}{27z} + \\
& \frac{16}{27}(1-2z)(1+2z)(3\log^2\sigma + 2\log\sigma\log(1-4z) - 3\log\sigma\log z + 4\text{Li}_2(\sigma) + \\
& 2\text{Li}_2(\sigma^2)) , \\
B_{3,22}^{s'} &= \frac{4\pi^2(1-z)}{3} - \frac{\sqrt{1-4z}(1129-2143z+186z^2)}{81} + \\
& \frac{2(21+78z-267z^2+90z^3+62z^4)\log\sigma}{27z} - \\
& \frac{8(2-5z)\sqrt{1-4z}\log x_1}{9} + \frac{112\sqrt{1-4z}(1-z)\log(1-4z)}{9} - \\
& \frac{4(1-z)(7+4z)\log\sigma\log(1-4z)}{9} - \frac{2\sqrt{1-4z}(7+40z-71z^2)\log z}{9z} - \\
& \frac{4(1-z)(5+2z)(\log^2\sigma - \log\sigma\log z)}{3} + \\
& \frac{16(1-2z)(1-z)\text{Li}_2(\sigma)}{9} - \frac{16(1-z)(4+z)\text{Li}_2(\sigma^2)}{9} , \\
B_{4,22}^{s'} &= \frac{-4\pi^2(1+2z)}{3} + \frac{\sqrt{1-4z}(691+1922z+1608z^2)}{81} - \\
& \frac{2(3+159z+138z^2-774z^3+536z^4)\log\sigma}{27z} + \\
& \frac{8\sqrt{1-4z}(1+2z)(2\log x_1 - 14\log(1-4z))}{9} +
\end{aligned}$$

$$\begin{aligned}
& \frac{4(1+2z)(7+4z)\log\sigma\log(1-4z)}{9} + \frac{2\sqrt{1-4z}(1+55z+154z^2)\log z}{9z} + \\
& \frac{4(1+2z)(5+2z)(\log^2\sigma - \log\sigma\log z)}{3} - \\
& \frac{16(1-2z)(1+2z)\text{Li}_2(\sigma)}{9} + \frac{16(4+z)(1+2z)\text{Li}_2(\sigma^2)}{9},
\end{aligned} \tag{41}$$

where  $\sigma$  is the ratio

$$\sigma = \frac{1 - \sqrt{1-4z}}{1 + \sqrt{1-4z}}, \tag{42}$$

and we have defined  $x_0 = \mu_0/m_b$  and  $x_1 = \mu_1/m_b$ .

Finally we present the results for the coefficients  $P_{k,ij}$  of the penguin and chromomagnetic operators defined in eq. (14). The coefficients  $P_{k,28}$  have been computed by using the convention in which the chromomagnetic coefficient  $C_{8G}$  has a positive sign. We obtain the expressions:

$$\begin{aligned}
P_{1,13} &= -\sqrt{1-4z}(1-z), & P_{1,23} &= \frac{-\sqrt{1-4z}(1-z)}{3}, & P_{1,14} &= \frac{-\sqrt{1-4z}(1-z)}{3}, \\
P_{2,13} &= \sqrt{1-4z}(1+2z), & P_{2,23} &= \frac{\sqrt{1-4z}(1+2z)}{3}, & P_{2,14} &= \frac{\sqrt{1-4z}(1+2z)}{3}, \\
P_{3,13} &= 0, & P_{3,23} &= 0, & P_{3,14} &= 0, \\
P_{4,13} &= 0, & P_{4,23} &= 0, & P_{4,14} &= 0.
\end{aligned} \tag{43}$$

$$\begin{aligned}
P_{1,24} &= \frac{-\sqrt{1-4z}(1-z)}{9}, & P_{1,15} &= -3z\sqrt{1-4z}, & P_{1,25} &= -z\sqrt{1-4z}, \\
P_{2,24} &= \frac{\sqrt{1-4z}(1+2z)}{9}, & P_{2,15} &= 0, & P_{2,25} &= 0, \\
P_{3,24} &= \frac{-2\sqrt{1-4z}(1-z)}{3}, & P_{3,15} &= 0, & P_{3,25} &= 0, \\
P_{4,24} &= \frac{2\sqrt{1-4z}(1+2z)}{3}, & P_{4,15} &= 0, & P_{4,25} &= 0.
\end{aligned} \tag{44}$$

$$\begin{aligned}
P_{1,16} &= -z\sqrt{1-4z}, & P_{1,26} &= \frac{-z\sqrt{1-4z}}{3}, & P_{1,28} &= 0, \\
P_{2,16} &= 0, & P_{2,26} &= 0, & P_{2,28} &= 0, \\
P_{3,16} &= 0, & P_{3,26} &= -2z\sqrt{1-4z}, & P_{3,28} &= \frac{-2\sqrt{1-4z}(1+2z)}{3}, \\
P_{4,16} &= 0, & P_{4,26} &= 0, & P_{4,28} &= \frac{2\sqrt{1-4z}(1+2z)}{3}.
\end{aligned} \tag{45}$$

Note that, in the limit of vanishing charm quark mass ( $z = 0$ ), the contribution of the penguin operators  $Q_5$  and  $Q_6$  vanish for chirality.

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